4/25/18 "Too many of us are not living our dreams because we are living our fears."-Les Brown

HW: "Angles" Finish Exercise Set A (skip #16-19)
Test 1 on Wednesday 5/2

AIM: What is an angle?

Warm Up:

MIT Topic: Trigonometry

Name: Key Date:

A2CC: More Practice with Sequences and Series

Do Now:

1. For some value of x the sequence $2x+1, x+11, 4x+5$ forms the first three terms of an arithmetic sequence.

(a) Find the value of x.

$x+11-(2x+1) = 4x+5-(x+11)$

$-x+10 = 3x-6$

$16 = 4x$

$\boxed{x=4}$

$2(4)+1 = 9$

$4+11 = 15$

$4(4)+5 = 21$

$\boxed{9, 15, 21}$

$15-9=6 \quad 21-15=6$

$d=6$

(b) Determine the numerical value of the 15th term of this sequence.

$a_{15} = 9+6(15-1)$

$a_{15} = 9+6(14)$

$a_{15} = \boxed{93}$

(c) Find the sum of the first 30 terms of this sequence. Show your analysis.

Need a_{30}

$a_{30} = 9+6(29)$

$a_{30} = \boxed{183}$

$S_{30} = \frac{30}{2}(9+183)$

$S_{30} = \boxed{2880}$

2. In a geometric sequence, the first term is -8 and the eighth term is 17,496. Determine the second term of this sequence. Show how you arrived at your result.

First $\rightarrow$ Eighth is 7 times

$a_8 = a_1 \cdot r^7$

$\frac{17496}{-8} = \frac{-8r^7}{-8}$

$-2187 = r^7$

$\sqrt[7]{-2187} = r$

$-3 = r$

$a_2 = a_1 \cdot r^1$

$a_2 = -8(-3)^1$

$\boxed{a_2 = 24}$

Practice

WORK SHOULD BE DONE ON A SEPARATE SHEET OF PAPER

For 1- 2: Find the first four terms as well as the tenth term of the sequence with the *n*th term.

1. $a_n = \frac{n^2}{n+1}$ $a_1 = \frac{1^2}{1+1} = \boxed{\frac{1}{2}}$ $a_2 = \frac{2^2}{2+1} = \boxed{\frac{4}{3}}$ $a_3 = \frac{3^2}{3+1} = \boxed{\frac{9}{4}}$ $a_4 = \frac{4^2}{4+1} = \boxed{\frac{16}{5}}$ $a_{10} = \frac{10^2}{10+1} = \boxed{\frac{100}{11}}$

2. $a_n = (-1)^n \frac{2^n}{n}$

For 3 and 4: A sequence is defined recursively. Find the first five terms of the sequence.

3. $a_n = a_{n-1} + 2n - 1,\ a_1 = 1$ $a_2 = a_1 + 2(2) - 1 = 1 + 4 - 1 = 4$ $a_3 = 4 + 2(3) - 1 = 9$ $a_4 = 9 + 2(4) - 1 = 16$ $a_5 = 16 + 2(5) - 1 = 25$ $\boxed{1, 4, 9, 16, 15}$

4. $a_{n+1} = a_n + 2a_{n-1},\ a_1 = 1,\ a_2 = 3$

For 5 – 7: The *n*th term of a sequence is given
(a) Find the first five terms of the sequence.
(b) Determine if the sequence is arithmetic or geometric. Find *d* or *r*.

5. $a_n = 2n + 5$ a) $a_1 = 2(1) + 5 = 7$, $a_2 = 2(2) + 5 = 9$, $a_3 = 2(3) + 5 = 11$, $a_4 = 2(4) + 5 = 13$, $a_5 = 2(5) + 5 = 15$ b) Arithmetic $d = 2$

6. $a_n = \frac{5}{2^n}$

7. $a_n = \frac{3^n}{2^{n+1}}$ $a_1 = \frac{3^1}{2^{1+1}} = \boxed{\frac{3}{4}}$ $a_2 = \frac{3^2}{2^{2+1}} = \boxed{\frac{9}{8}}$ $a_3 = \frac{3^3}{2^{3+1}} = \boxed{\frac{27}{16}}$ $a_4 = \frac{3^4}{2^{4+1}} = \boxed{\frac{81}{32}}$ $a_5 = \frac{3^5}{2^{5+1}} = \boxed{\frac{243}{64}}$

b) $\frac{\frac{9}{8}}{\frac{3}{4}} = \frac{3}{2}$ $\frac{\frac{27}{16}}{\frac{9}{8}} = \frac{3}{2}$ b) Geometric $r = \frac{3}{2}$

For 8 – 10: The first four terms of a sequence are given. Determine whether they can be the terms of an arithmetic sequence, a geometric sequence, or neither. If the sequence is arithmetic or geometric, find the fifth term.

8. $5, 5.5, 6, 6.5, \ldots$

9. $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, \ldots$ $2\sqrt{2}-\sqrt{2}=\sqrt{2}$, $3\sqrt{2}-2\sqrt{2}=\sqrt{2}$, $4\sqrt{2}-3\sqrt{2}=\sqrt{2}$ Arithmetic

10. $\sqrt{2}, 2, 2\sqrt{2}, 4, \ldots$

11. Show that $3, 6i, -12, -24i, \ldots$ is a geometric sequence, and find the common ratio.
$\frac{-24i}{-12} = 2i$ ✓ $\frac{6i}{3} = 2i$ ✓ $r = 2i$

12. The sixth term of an arithmetic sequence is 17, and the fourth term is 11. Find the second term.

13. The 20th term of an arithmetic sequence is 96, and the common difference is 5. Find the nth term. $a_n = a_1 + d(n-1)$ (need a_1) To find a_1: $96 = a_1 + 5(20-1) \rightarrow a_1 = 1$, $96 = a_1 + 95$ $a_n = 1 + 5(n-1)$

14. The third term of a geometric sequence is 9, and the common ratio is $\frac{3}{2}$. Find the fifth term.

15. The second term of a geometric sequence is 10, and the fifth term is $\frac{1250}{27}$. Find the nth term.

10 _ _ _ $\frac{1250}{27}$

$10 = a_1 \cdot r^{2-1}$, $10 = a_1 \cdot r^1$, $10 = a_1 \cdot \frac{5}{3}$, $\frac{10}{5/3} = a_1$, $6 = a_1$

$\frac{1250}{27} = 10r^3$, $\frac{125}{27} = r^3$, $\frac{5}{3} = r$

$a_n = 6\left(\frac{5}{3}\right)^{n-1}$

16. Evaluate: $\sum_{k=3}^{6}(k+1)^3$

$C_K = 100\left(\frac{1}{2}\right)^{K-1}$ $\sum_{K=1}^{10} 100\left(\frac{1}{2}\right)^{K-1}$ $C_1 = 100$ $C_2 = 100 \cdot \frac{1}{2}$ $C_3 = 100 \cdot \frac{1}{2} \cdot \frac{1}{2}$ $C_4 = 100\left(\frac{1}{2}\right)^3$ $C_5 = 100\left(\frac{1}{2}\right)^4$

17. If a sequence is defined by the recursive formula:

$$c_1 = 100 \text{ and } c_i = c_{i-1} \cdot \frac{1}{2}$$

then what is the value of $\sum_{k=1}^{10} c_i$. Show how you arrived at your answer.

Sum of first 10 terms

$\frac{25575}{128}$

For 18 and 19: Write the sum using sigma notation. Do not evaluate.

18. $3+6+9+12+\ldots+99$

19. $1^2+2^2+3^2+4^2+\ldots+100^2$ $\sum_{x=1}^{100} x^2$

20. The first term of an arithmetic sequence is 7, and the common difference is 3. How many terms of this sequence must be added to obtain 325?

21. The sum of the first three terms of a geometric series is 52, and the common ratio is 3. Find the first term.

n

r

$52 = \frac{a_1(1-3^3)}{1-3} = \frac{a_1(1-27)}{-2} = \frac{a_1(-26)}{-2} = 13a_1$

$\frac{52}{13} = \frac{13a_1}{13}$

$a_1 = 4$

22. Find the number of terms in the sequence 8,13,18,....133.

23. The first term of an arithmetic sequence is 7, and the common difference is -4. Is -4,989 a term of the sequence? If so, which term is it?

yes

$-4989 = 7 - 4(n-1)$

$-4996 = -4(n-1)$

$1249 = (n-1)$

$+1 \quad +1$

$1250 = n$

24. How many terms are in the arithmetic sequence 5, 11, 17, … if the sum of the terms of the sequence is 1496?

25. Which term of the geometric sequence 2, 6, 18, … is 1,062,882?

$\times 3 \quad \times 3 \quad r = 3$

$\frac{1062882}{2} = \frac{2(3)^{n-1}}{2}$

$531441 = 3^{n-1}$

$n - 1 = \log_3 531441$

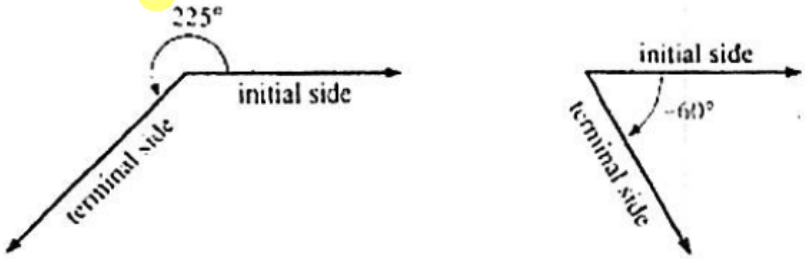
$+1 \qquad +1$

$n = \log_3 531441 + 1$

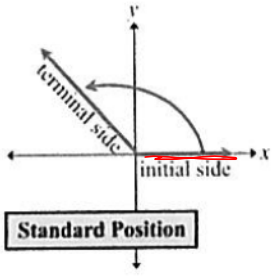
$n = 13$

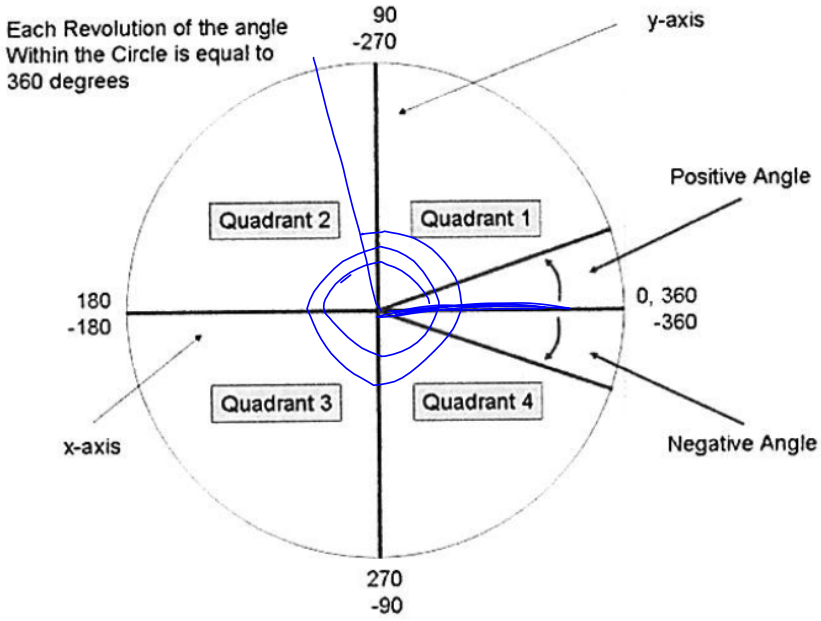
$a_n = a_1 r^{n-1}$

Two rays that have the same endpoint form an angle. One ray is fixed while the other ray is allowed to move around the endpoint. The endpoint of the rays is called the ***vertex*** of the angle. The fixed ray is the initial side of the angle, and the rotated ray is the terminal side. *Counterclockwise* rotations produce positive angles and *clockwise* rotations produce negative angles.



An angle in ***standard position*** has its vertex at the origin of a coordinate system and the initial side of the angle coincides with the positive x- axis. Depending on whether the terminal side falls in Quadrant I, II, III, or IV, we say that the angles lies in the first, second, third or fourth quadrant.





θ = "Theta" = angle measure

An angle whose terminal side coincides with a coordinate axis is called a ***quadrantal angle***.

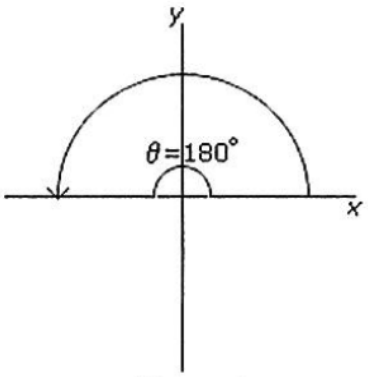


Figure 1

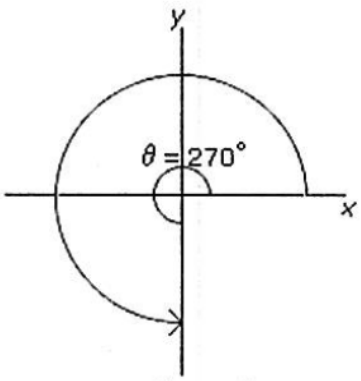
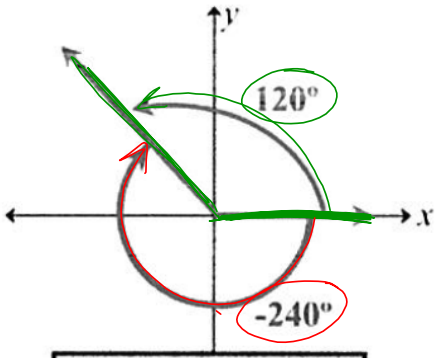


Figure 2

Angles sharing the same initial and terminal sides are called ***coterminal angles***.



Coterminal Angles

120° and -240° are coterminal

To find coterminal angles: Add or subtract 360°.

If two angles are coterminal, their difference is : A multiple of 360°.

(*) Subtract the angles.

Exercise Set A

For 1 – 19, determine in which quadrant the angle of the given measure lies.

1. 215° QIII
2. -110°
3. 318°
4. 72° QI
5. 95°
6. -45°
7. 225° QIII
8. 150°
9. 422°
10. -240° QII
11. 680°
12. 23°
13. 812° $812 - 360 = 452$, $452 - 360 = 92$ QII
14. -300°
15. 289°

For 20 - 27, name the least possible positive measure and the greatest possible negative measure of an angle that is coterminal with the given angle.

20. 70°

$70 + 360 = \boxed{430^\circ}$

$70 - 360 = \boxed{-290}$

21. -60°

$-60 + 360 = \boxed{300^\circ}$

$-60 - 360 = \boxed{-420^\circ}$

26. -180°

27. -930°

$-930 + 360 = -570 + 360 = \boxed{-210^\circ}$

$-210 + 360 = \boxed{150^\circ}$

For 28- 31, determine whether the following pairs of angles in standard position are coterminal.

28. 40° and 400° ✓

$400 - 40 = 360$

29. -120° and 120° Not coterminal

$120 - (-120) = 240$

Not a multiple of 360°

For 32 - 41, sketch an angle in standard position with the given measure

32. 100° 33. 360° 34. 90°

41. 1140°